# GW Detection Techniques

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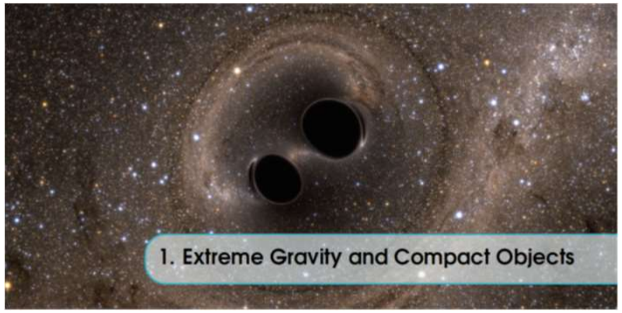
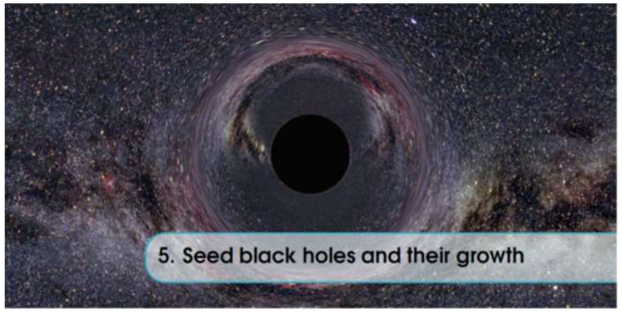
Credits to Jonah Kanner and Peter Shawhan for some slides

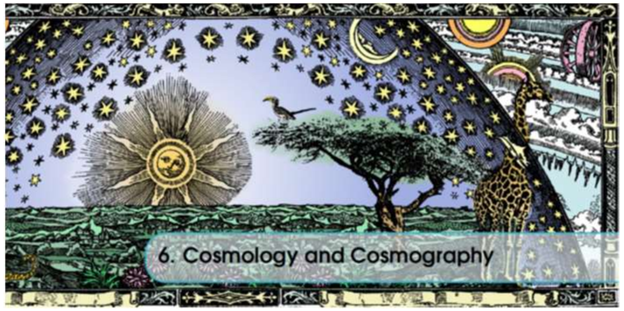
# A New Window to the Universe











Science with the current and future generation ground-based GW detectors

# GW Analysis Tutorials

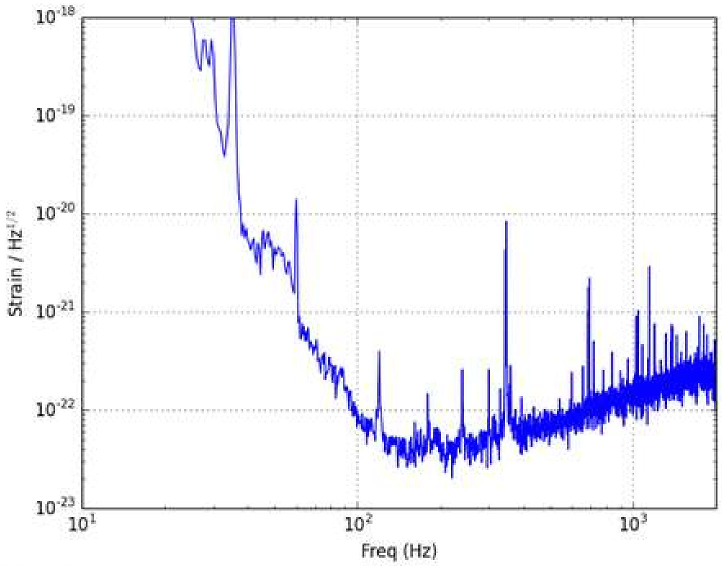
www.gw-openscience.org

**Amplitude Spectral Density (ASD)**

The ASD can be obtained by taking the square root of the PSD. This is done to give units that can be more easily compared with the time domain data or FFT. More information about the LIGO ASD can be seen on the Instrumental Lines page.

Learn how to:

- Read LIGO/Virgo data
- Plots with Python
- PSDs, FFTs, and more

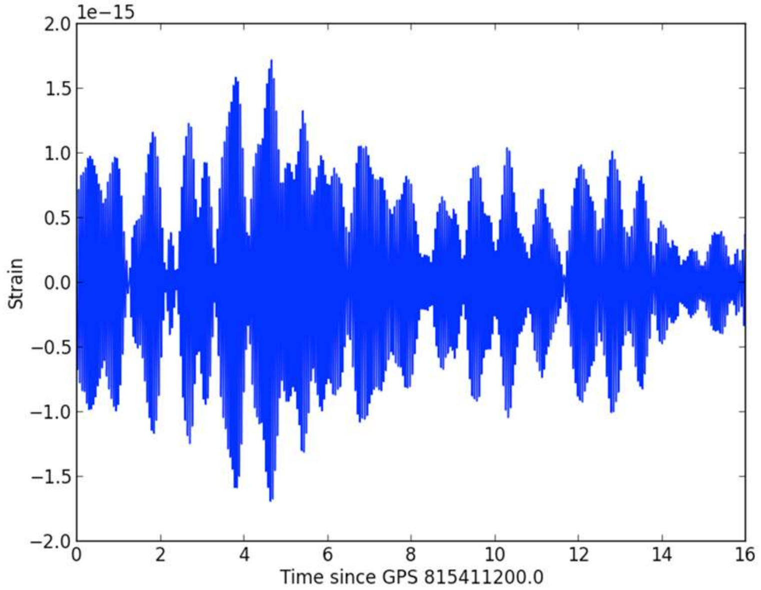



Show the code

```
#-------------------------
# Plot the ASD
#-------------------------------
Pxx, freqs = mlab.psd(strain_seg, Fs=fs, NFFT=2*fs)
plt.loglog(freqs, np.sqrt(Pxx))
plt.axis([10, 2000, 1e-23, 1e-18])
plt.grid('on')
plt.xlabel('Freq (Hz)')
plt.ylabel('Strain / Hz$^{1/2}$')
```

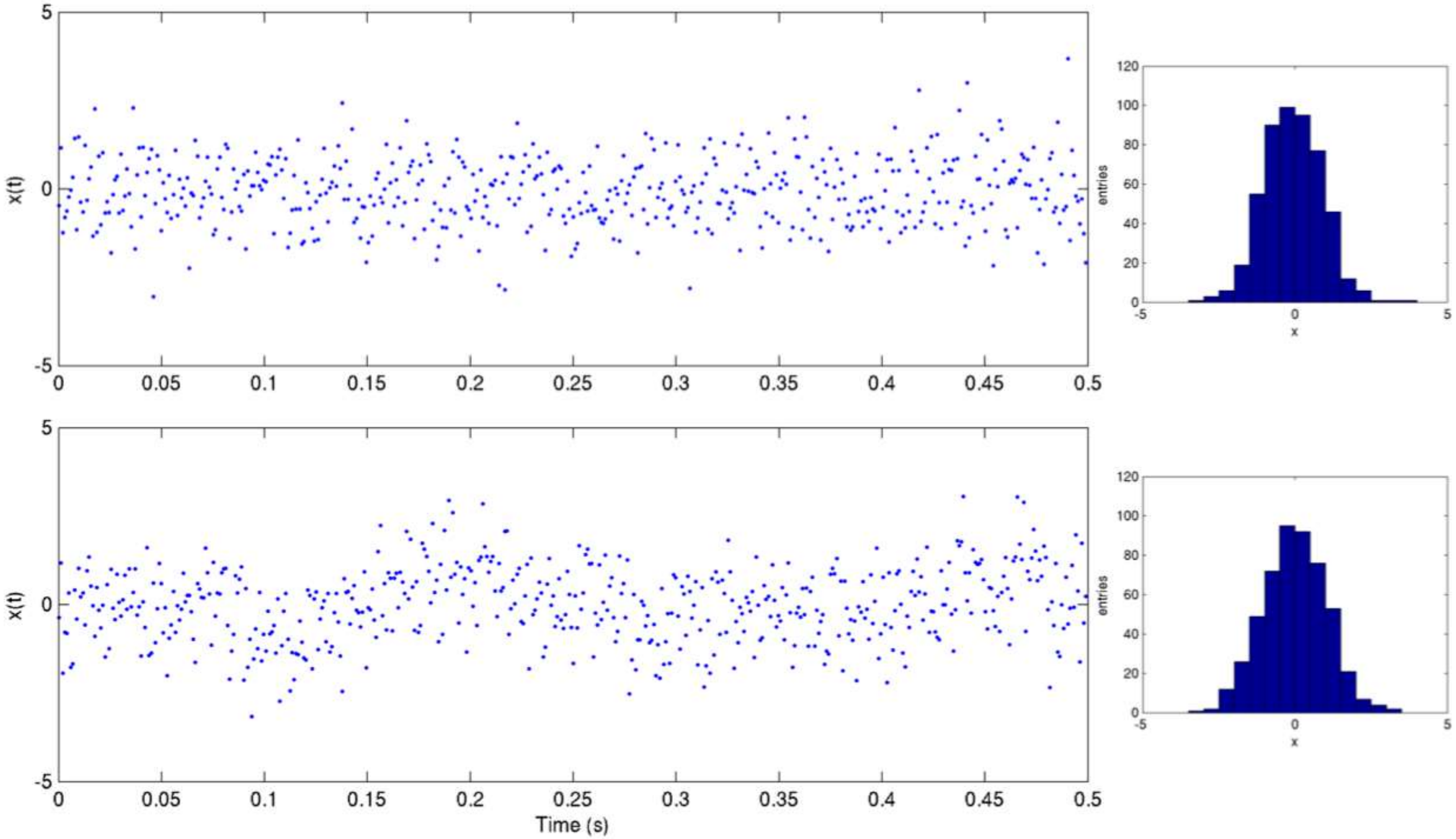
# Virgo Data

- Discretely sampled time-series data
- h(t) – calibrated strain
  - ALSO: hundreds of “auxiliary” channels
- h(t) recorded at 20 kHz sample rate
- ~100 MB per hour
- Stored in .gwf “frame” files



# Characterization of Random Noise

**Noise is random, but its *properties* can be characterized**



# Possible Properties of Noise

**Stationary : statistical properties are independent of time**

Ergodic process: time averages are equivalent to ensemble averages

**Gaussian : A random variable follows Gaussian distribution**

For a single random variable, $p(x) = \frac{1}{\sqrt{2\pi\sigma_x^2}} \exp\left[-\frac{1}{2}\frac{(x-\mu_x)^2}{\sigma_x^2}\right]$

More generally, a *set* of random variables (e.g. a time series) is Gaussian if the joint probability distribution is governed by a covariance matrix

$$C_{xij} := \langle x_i x_j \rangle - \langle x_i \rangle \langle x_j \rangle$$

such that

$$p(x_1, x_2, \ldots, x_N) = \frac{1}{(2\pi)^{N/2}\sqrt{\det C_x}} \exp\left[-\frac{1}{2}\sum_{i,j=0}^{N-1} C_{xij}^{-1}(x_i - \mu_{xi})(x_j - \mu_{xj})\right]$$

**White : Signal power is uniformly distributed over frequency**

⇒ Data samples are uncorrelated

# Fourier Transform represents data in the frequency domain

**Fourier transform**

$$\widetilde{x}(f) = \int_{-\infty}^{\infty} dt\, x(t) e^{-i2\pi f t}$$

$$\Rightarrow \qquad x(t) = \int_{-\infty}^{\infty} df\, \widetilde{x}(f) e^{i2\pi f t}$$

$|\widetilde{x}(f)|^2$ **can be interpreted as energy spectral density**

Efficient way to calculate complete discrete Fourier Transform:
Fast Fourier Transform (FFT)

# Power Spectral Density

**Parseval's theorem:**

$$\int_{-\infty}^{\infty} dt\, |x(t)|^2 = \int_{-\infty}^{\infty} df\, |\widetilde{x}(f)|^2$$

$\Rightarrow$ Total energy in the data can be calculated in either time domain or frequency domain

$|\widetilde{x}(f)|^2$ can be interpreted as energy spectral density

**When noise (or signal) has infinite extent in time domain, can still define the power spectral density (PSD)**

$$\lim_{T\to\infty} \frac{1}{T} \left|\widetilde{x}_T(f)\right|^2$$

**Watch out for one-sided vs. two-sided PSDs**

# Estimating the PSD

**Simplest approach: FFT the data, calculate square of magnitude of each frequency component – this is a periodogram**

For stationary noise, one can show that the frequency components are statistically independent

**This estimate is unbiased (has the correct mean), but has a large variance – so average several periodograms**

Alternately, smooth periodogram; give up frequency resolution either way

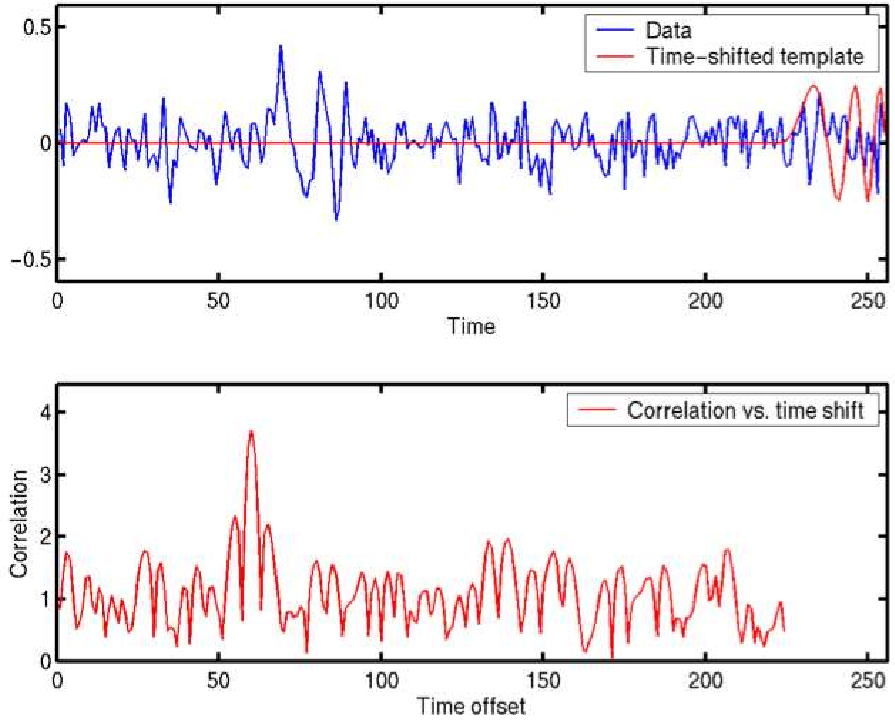
**Generally apply a "window" to the data to avoid spectral leakage**

Leakage arises from the assumption that the data is periodic!

Tapered window forces data to go to zero at ends of time interval

**Welch's method of estimating a PSD averages periodograms calculated from windowed data**

# Cross correlation
# Slide template against data




$$C(t) = \int_{-\infty}^{\infty} dt' \, s(t') \, h(t' - t)$$

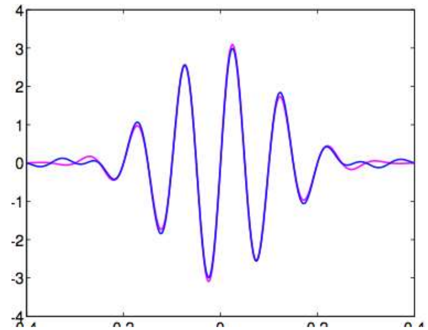
Time offset

Data

Template with time offset

# Burst searches look for excess power

$$X(\tau,\phi,Q) = \int_{-\infty}^{+\infty} x(t)\, w(t-\tau,\phi,Q)\, e^{-i2\pi\phi t}\, dt,$$




"Transform" to a time-frequency basis:

Cross-correlate data with wavelets to get energy in each time-frequency pixel

Similar to how Fourier Transform cross correlates against sine waves to get energy in each frequency bin

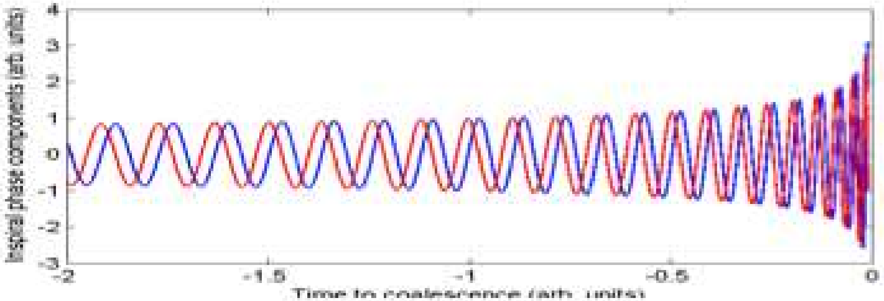
# CBC: Compact Binary Coalescence

**Inspiral source parameters**

- Masses (m1, m2)
- Spins → Negligible for neutron stars, at least
- Orbital phase at coalescence → Maximize analytically when filtering
- Inclination of orbital plane
- Sky location

  → Simply multiplicative for a given detector (long-wavelength limit)
- Distance → Simply multiplicative
- Coalescence time

Filter with orthogonal templates, take quadrature sum




➔ Only have to explicitly search over masses and coalescence time ("intrinsic parameters")

# Matched Filtering in the frequency domain

$$C(t) = \int_{-\infty}^{\infty} dt' \, s(t') \, h(t'-t)$$

Time offset

Data

Template with time offset

**Rewrite correlation integral using Fourier transforms…**

$$\Rightarrow \quad C(t) = 4 \int_{0}^{\infty} \tilde{s}(f) \, \tilde{h}^{*}(f) \, e^{2\pi i f t} \, df$$

Correlate in the frequency domain

“Phase factor” represents the time offsets

# Optimal Matched Filtering

FFT of data

Template; can be generated in frequency domain using stationary phase approximation

$$C(t) = 4 \int_0^{\infty} \frac{\tilde{s}(f)\ \tilde{h}^*(f)}{S_n(f)}\ e^{2\pi i f t}\ df$$

Noise power spectral density

**Look for maximum of $|C(t)|$ above some threshold → trigger**

Putting the noise PSD in the denominator "down weights" places where the noise is high

# CBC Template Banks

**Want to be able to detect any signal in a *space* of possible signals**

All with different phase evolution

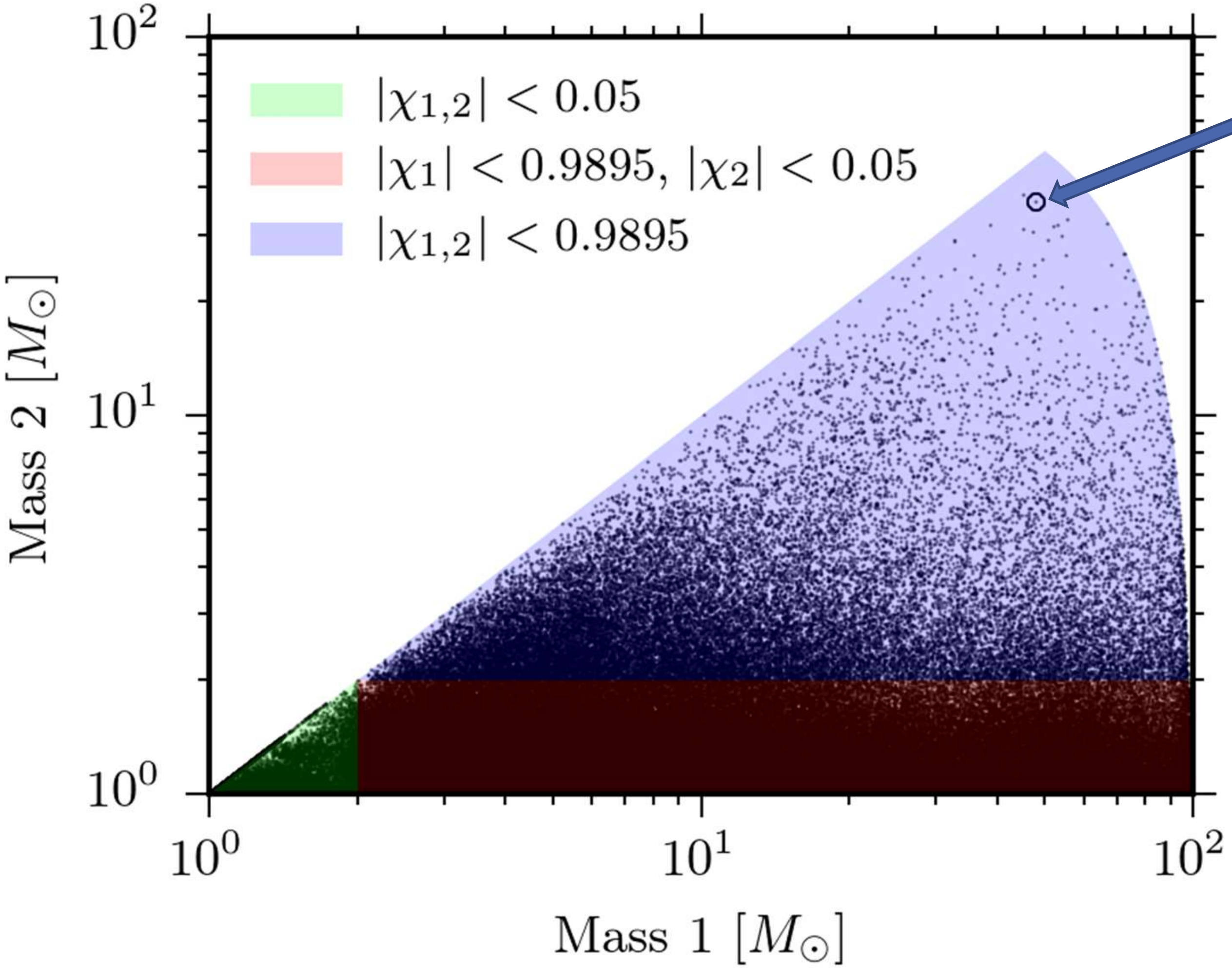
**… but do it with a finite set of templates!**

**So make sure there is a "close enough" template for every part of the signal space**

Require a minimum overlap between signal and template, e.g. 0.97

**Often can calculate a "metric" which parametrizes the mismatch for small mismatches**

# CBC Template Banks in "Mass Space"

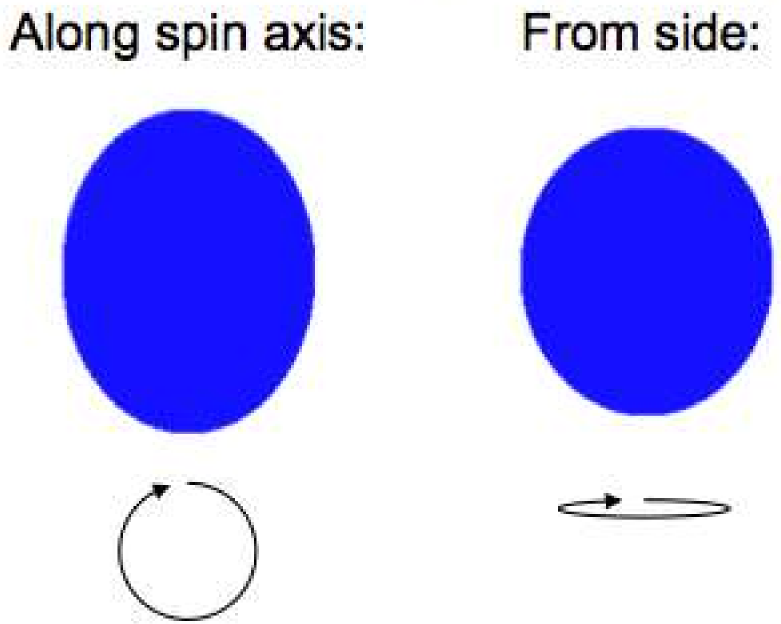



Best match with GW150914

# Continuous Wave sources from spinning neutron stars

**If not axisymmetric, will emit gravitational waves**

**Example: ellipsoid with distinct transverse axes**



# Continuous Wave signals

**Start with a sinusoidal signal with spin-down term(s)**

Polarization content depends on orientation/inclination of spin axis

**Amplitude modulation**

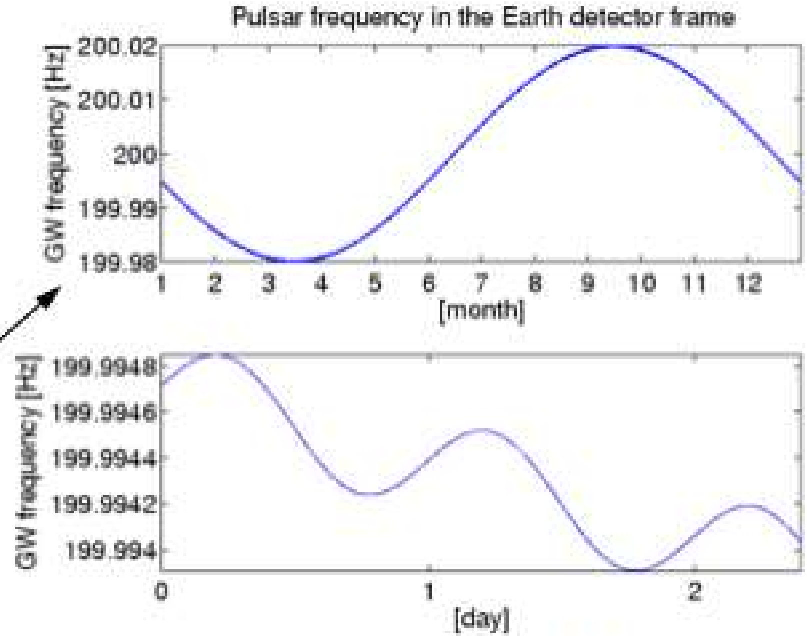
Polarization projection changes over a sidereal day

**Doppler shift**

$$\frac{\Delta f}{f} = \frac{\mathbf{v} \cdot \mathbf{n}}{c}$$

Annual variation: up to $\sim 10^{-4}$

Daily variation: up to $\sim 10^{-6}$




**GW signals from binary systems are more complicated !**

Additional Doppler shift due to orbital motion of neutron star

Varying gravitational redshift if orbit is elliptical

Shapiro time delay if GW passes near companion

# CW – Wide parameter space

**Method: matched filtering with a bank of templates**

**Parameters:**

- Sky position
- Spin axis inclination and azimuthal angle
- Frequency, spindown, initial phase
- Binary orbit parameters (if in a binary system)

**Can use a detection statistic, $\mathcal{F}$, which analytically maximizes over spin axis inclination & azimuthal angle and initial phase**

- Even so, computing cost scales as $\sim T^6$
- Detection threshold also must increase with number of templates

**Check for signal consistency in multiple detectors**

**Problem: huge number of templates needed**

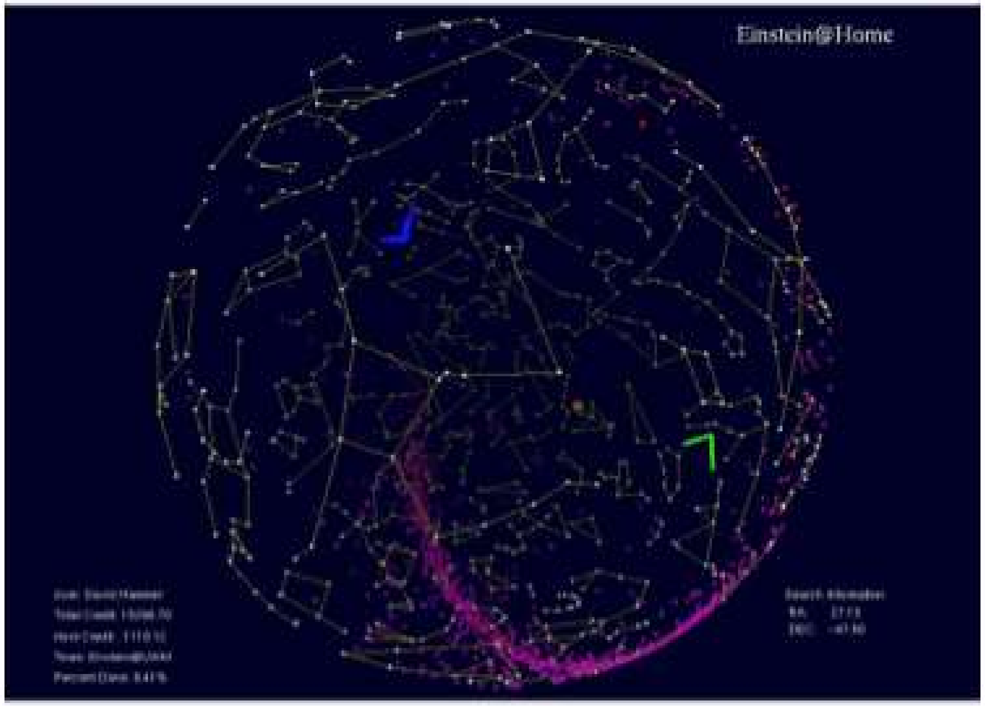
- Even using clever semi-coherent analysis methods

# Citizen Science

**Public distributed computing project: Einstein@Home**

einsteinathome.org

Small bits of data distributed for processing;
results collected, verified, and post-processed




Searching for CW signals in LIGO+Virgo data

Also searching for millisecond pulsars in data from Arecibo, Parkes, and the Fermi satellite

# Stochastic Signals

**Random signal from sum of unresolved sources**

From the early universe, or from astrophysical sources since then

**Usual assumptions about the signal:**

Stationary

Gaussian

Unpolarized

Power-law frequency dependence, probably (e.g. $f^{-3}$)

**May be isotropic, or not**

**Looks basically like extra noise in each detector !**

# Use cross-correlation to find stochastic signals

**Use cross-correlation between GW data streams**

No time delay for all-sky isotropic search – will affect correlation

For anisotropic ("radiometer") search, fix time delay between streams

**Include a filter function in the cross-correlation**

$$Y := \int_{-T/2}^{T/2} dt_1 \int_{-T/2}^{T/2} dt_2\, x_1(t_1) x_2(t_2) K(t_1 - t_2)$$

$$\Downarrow$$

$$Y = \int_{-\infty}^{\infty} df \int_{-\infty}^{\infty} df'\, \delta_T(f - f') \tilde{x}_1^*(f) \tilde{x}_2(f') \widetilde{K}(f')$$

**Filter function optimizes the detection statistic, accounting for two effects:**

Power spectrum of the signal being searched for

Expected correlation between detectors, which depends on frequency due to their separation

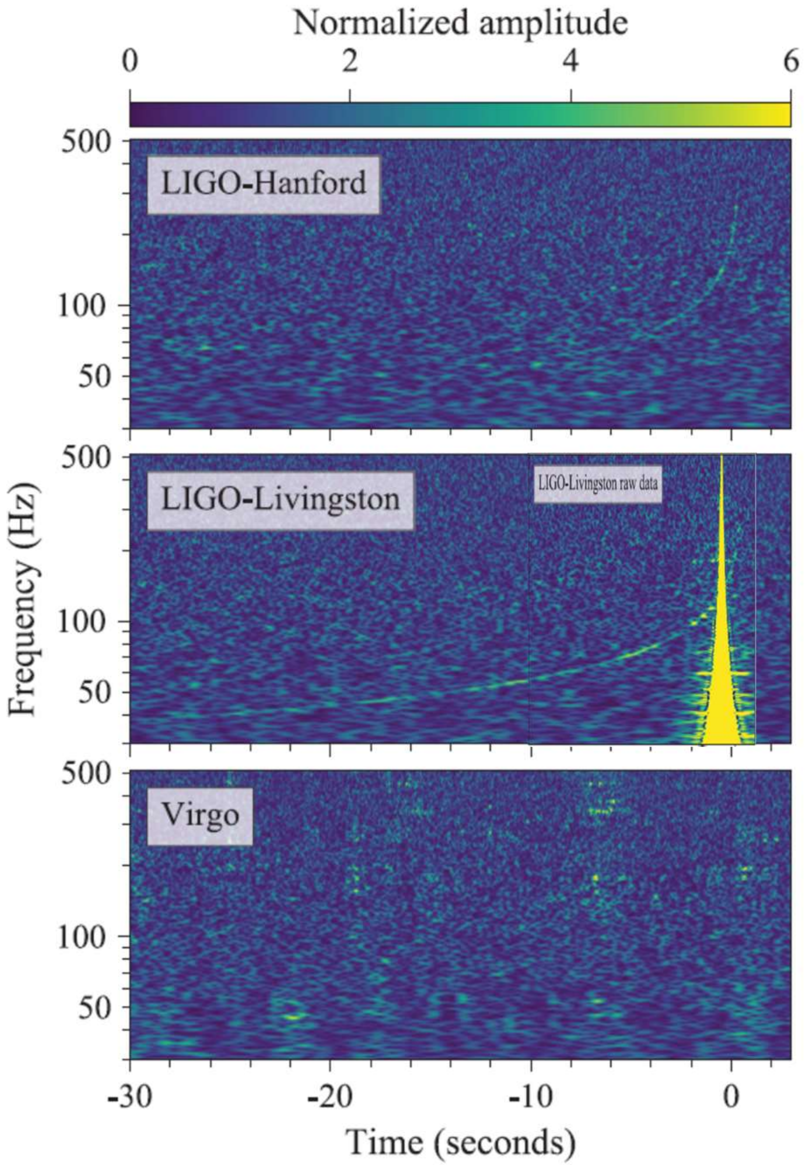
# Cross-correlations are used in all types of searches

- CBC searches cross-correlate data against a template bank of merging NS-NS or BH-BH
- Burst searches cross-correlate against wavelets to transform to time-frequency space
- Continuous Wave searches cross-correlate against sine-waves with doppler shifts
- Stochastic searches cross-correlate data from 2 detectors

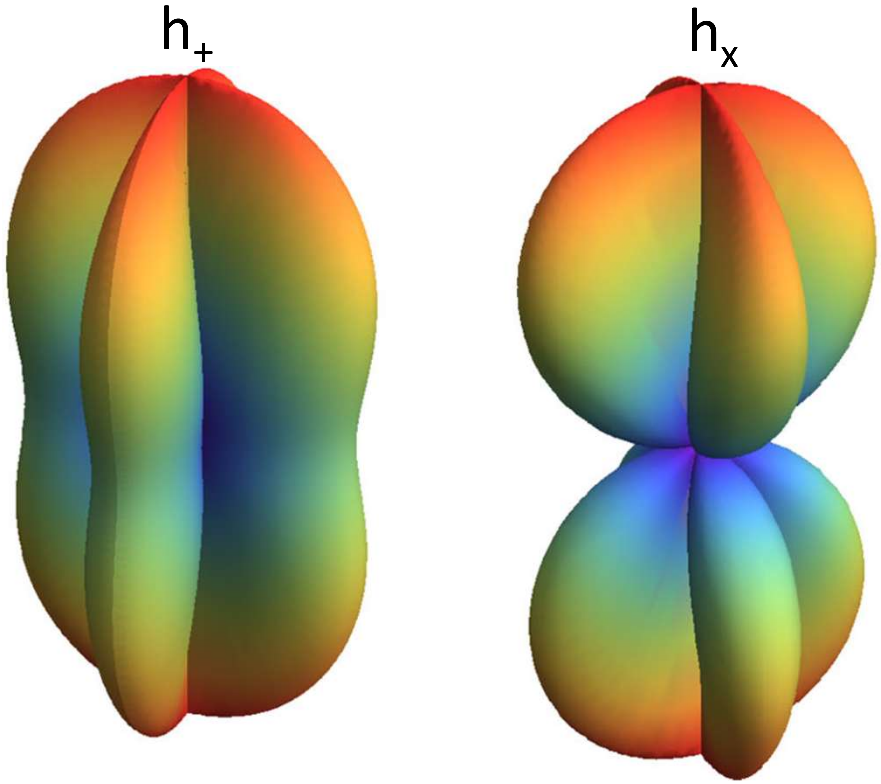
# GWTC-1

| Event | $m_1/\mathrm{M}_\odot$ | $m_2/\mathrm{M}_\odot$ | $\mathcal{M}/\mathrm{M}_\odot$ | $\chi_{\mathrm{eff}}$ | $M_{\mathrm{f}}/\mathrm{M}_\odot$ | $a_{\mathrm{f}}$ | $E_{\mathrm{rad}}/(\mathrm{M}_\odot c^2)$ | $\ell_{\mathrm{peak}}/(\mathrm{erg\,s^{-1}})$ | $d_L/\mathrm{Mpc}$ | $z$ | $\Delta\Omega/\mathrm{deg}^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| GW150914 | $35.6^{+4.8}_{-3.0}$ | $30.6^{+3.0}_{-4.4}$ | $28.6^{+1.6}_{-1.5}$ | $-0.01^{+0.12}_{-0.13}$ | $63.1^{+3.3}_{-3.0}$ | $0.69^{+0.05}_{-0.04}$ | $3.1^{+0.4}_{-0.4}$ | $3.6^{+0.4}_{-0.4} \times 10^{56}$ | $430^{+150}_{-170}$ | $0.09^{+0.03}_{-0.03}$ | 180 |
| GW151012 | $23.3^{+14.0}_{-5.5}$ | $13.6^{+4.1}_{-4.8}$ | $15.2^{+2.0}_{-1.1}$ | $0.04^{+0.28}_{-0.19}$ | $35.7^{+9.9}_{-3.8}$ | $0.67^{+0.13}_{-0.11}$ | $1.5^{+0.5}_{-0.5}$ | $3.2^{+0.8}_{-1.7} \times 10^{56}$ | $1060^{+540}_{-480}$ | $0.21^{+0.09}_{-0.09}$ | 1555 |
| GW151226 | $13.7^{+8.8}_{-3.2}$ | $7.7^{+2.2}_{-2.6}$ | $8.9^{+0.3}_{-0.3}$ | $0.18^{+0.20}_{-0.12}$ | $20.5^{+6.4}_{-1.5}$ | $0.74^{+0.07}_{-0.05}$ | $1.0^{+0.1}_{-0.2}$ | $3.4^{+0.7}_{-1.7} \times 10^{56}$ | $440^{+180}_{-190}$ | $0.09^{+0.04}_{-0.04}$ | 1033 |
| GW170104 | $31.0^{+7.2}_{-5.6}$ | $20.1^{+4.9}_{-4.5}$ | $21.5^{+2.1}_{-1.7}$ | $-0.04^{+0.17}_{-0.20}$ | $49.1^{+5.2}_{-3.9}$ | $0.66^{+0.08}_{-0.10}$ | $2.2^{+0.5}_{-0.5}$ | $3.3^{+0.6}_{-0.9} \times 10^{56}$ | $960^{+430}_{-410}$ | $0.19^{+0.07}_{-0.08}$ | 924 |
| GW170608 | $10.9^{+5.3}_{-1.7}$ | $7.6^{+1.3}_{-2.1}$ | $7.9^{+0.2}_{-0.2}$ | $0.03^{+0.19}_{-0.07}$ | $17.8^{+3.2}_{-0.7}$ | $0.69^{+0.04}_{-0.04}$ | $0.9^{+0.05}_{-0.1}$ | $3.5^{+0.4}_{-1.3} \times 10^{56}$ | $320^{+120}_{-110}$ | $0.07^{+0.02}_{-0.02}$ | 396 |
| GW170729 | $50.6^{+16.6}_{-10.2}$ | $34.3^{+9.1}_{-10.1}$ | $35.7^{+6.5}_{-4.7}$ | $0.36^{+0.21}_{-0.25}$ | $80.3^{+14.6}_{-10.2}$ | $0.81^{+0.07}_{-0.13}$ | $4.8^{+1.7}_{-1.7}$ | $4.2^{+0.9}_{-1.5} \times 10^{56}$ | $2750^{+1350}_{-1320}$ | $0.48^{+0.19}_{-0.20}$ | 1033 |
| GW170809 | $35.2^{+8.3}_{-6.0}$ | $23.8^{+5.2}_{-5.1}$ | $25.0^{+2.1}_{-1.6}$ | $0.07^{+0.16}_{-0.16}$ | $56.4^{+5.2}_{-3.7}$ | $0.70^{+0.08}_{-0.09}$ | $2.7^{+0.6}_{-0.6}$ | $3.5^{+0.6}_{-0.9} \times 10^{56}$ | $990^{+320}_{-380}$ | $0.20^{+0.05}_{-0.07}$ | 340 |
| GW170814 | $30.7^{+5.7}_{-3.0}$ | $25.3^{+2.9}_{-4.1}$ | $24.2^{+1.4}_{-1.1}$ | $0.07^{+0.12}_{-0.11}$ | $53.4^{+3.2}_{-2.4}$ | $0.72^{+0.07}_{-0.05}$ | $2.7^{+0.4}_{-0.3}$ | $3.7^{+0.4}_{-0.5} \times 10^{56}$ | $580^{+160}_{-210}$ | $0.12^{+0.03}_{-0.04}$ | 87 |
| GW170817 | $1.46^{+0.12}_{-0.10}$ | $1.27^{+0.09}_{-0.09}$ | $1.186^{+0.001}_{-0.001}$ | $0.00^{+0.02}_{-0.01}$ | $\leq 2.8$ | $\leq 0.89$ | $\geq 0.04$ | $\geq 0.1 \times 10^{56}$ | $40^{+10}_{-10}$ | $0.01^{+0.00}_{-0.00}$ | 16 |
| GW170818 | $35.5^{+7.5}_{-4.7}$ | $26.8^{+4.3}_{-5.2}$ | $26.7^{+2.1}_{-1.7}$ | $-0.09^{+0.18}_{-0.21}$ | $59.8^{+4.8}_{-3.8}$ | $0.67^{+0.07}_{-0.08}$ | $2.7^{+0.5}_{-0.5}$ | $3.4^{+0.5}_{-0.7} \times 10^{56}$ | $1020^{+430}_{-360}$ | $0.20^{+0.07}_{-0.07}$ | 39 |
| GW170823 | $39.6^{+10.0}_{-6.6}$ | $29.4^{+6.3}_{-7.1}$ | $29.3^{+4.2}_{-3.2}$ | $0.08^{+0.20}_{-0.22}$ | $65.6^{+9.4}_{-6.6}$ | $0.71^{+0.08}_{-0.10}$ | $3.3^{+0.9}_{-0.8}$ | $3.6^{+0.6}_{-0.9} \times 10^{56}$ | $1850^{+840}_{-840}$ | $0.34^{+0.13}_{-0.14}$ | 1651 |

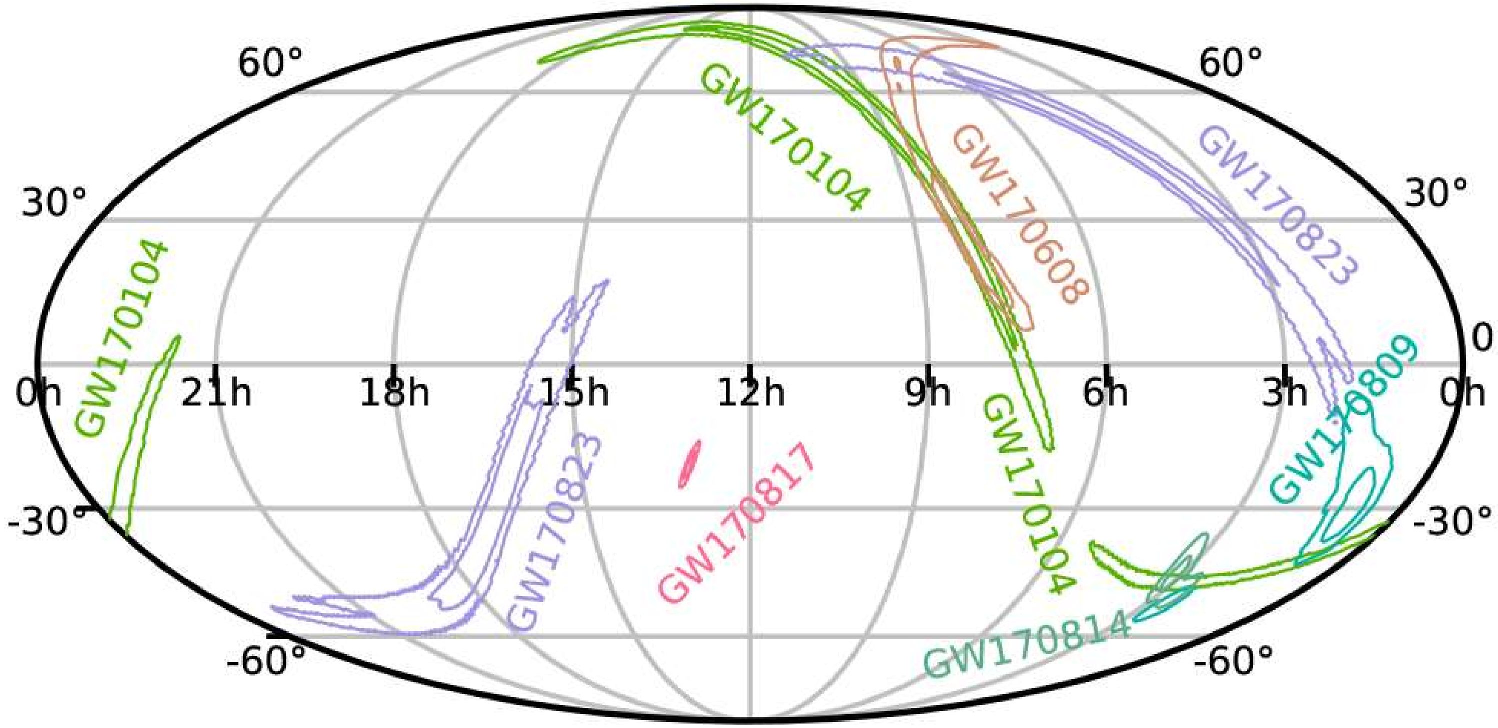
# Detection of GW170817




Spectrograms of GW170817 in the three detectors.

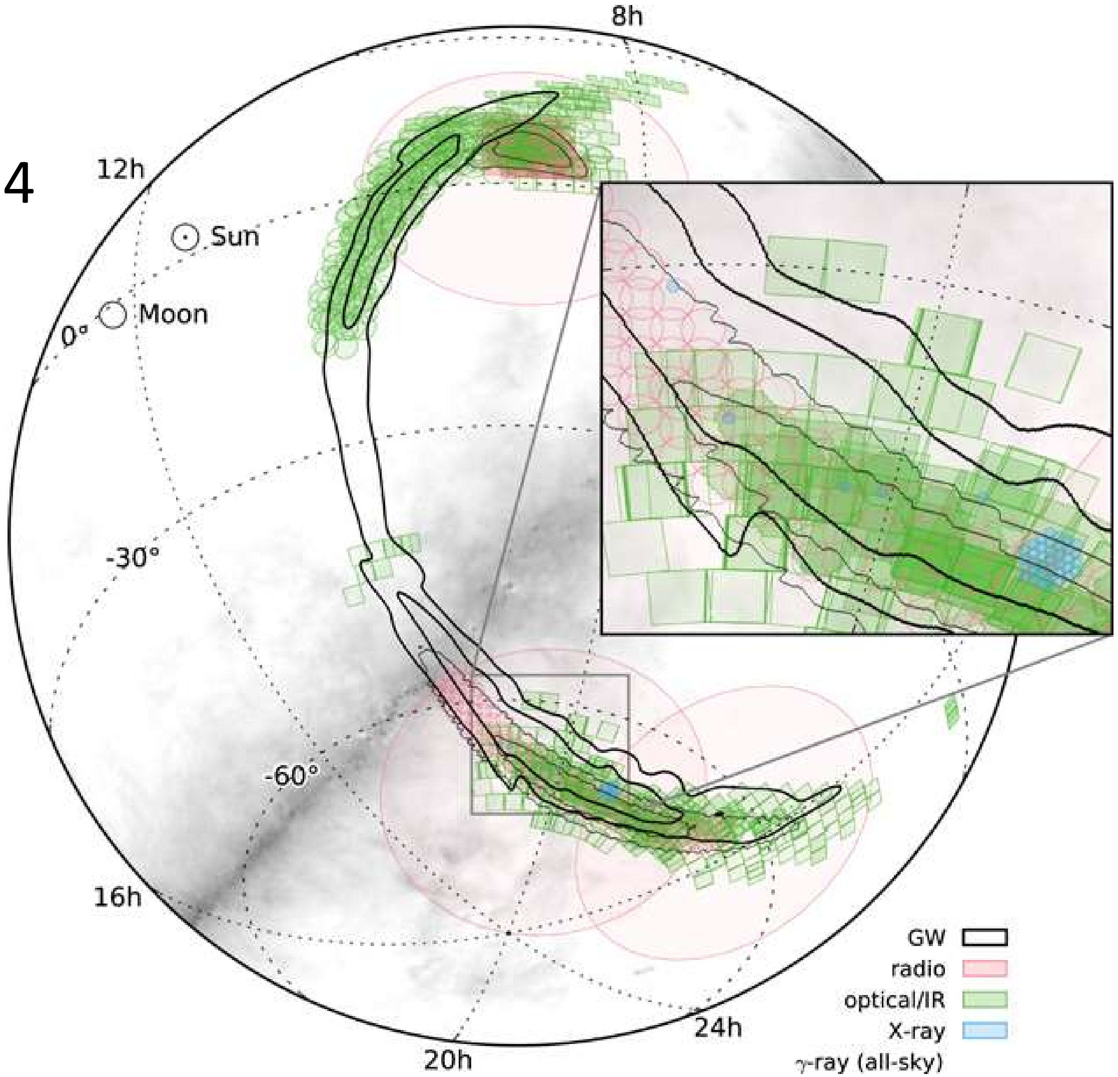


# Source Localization

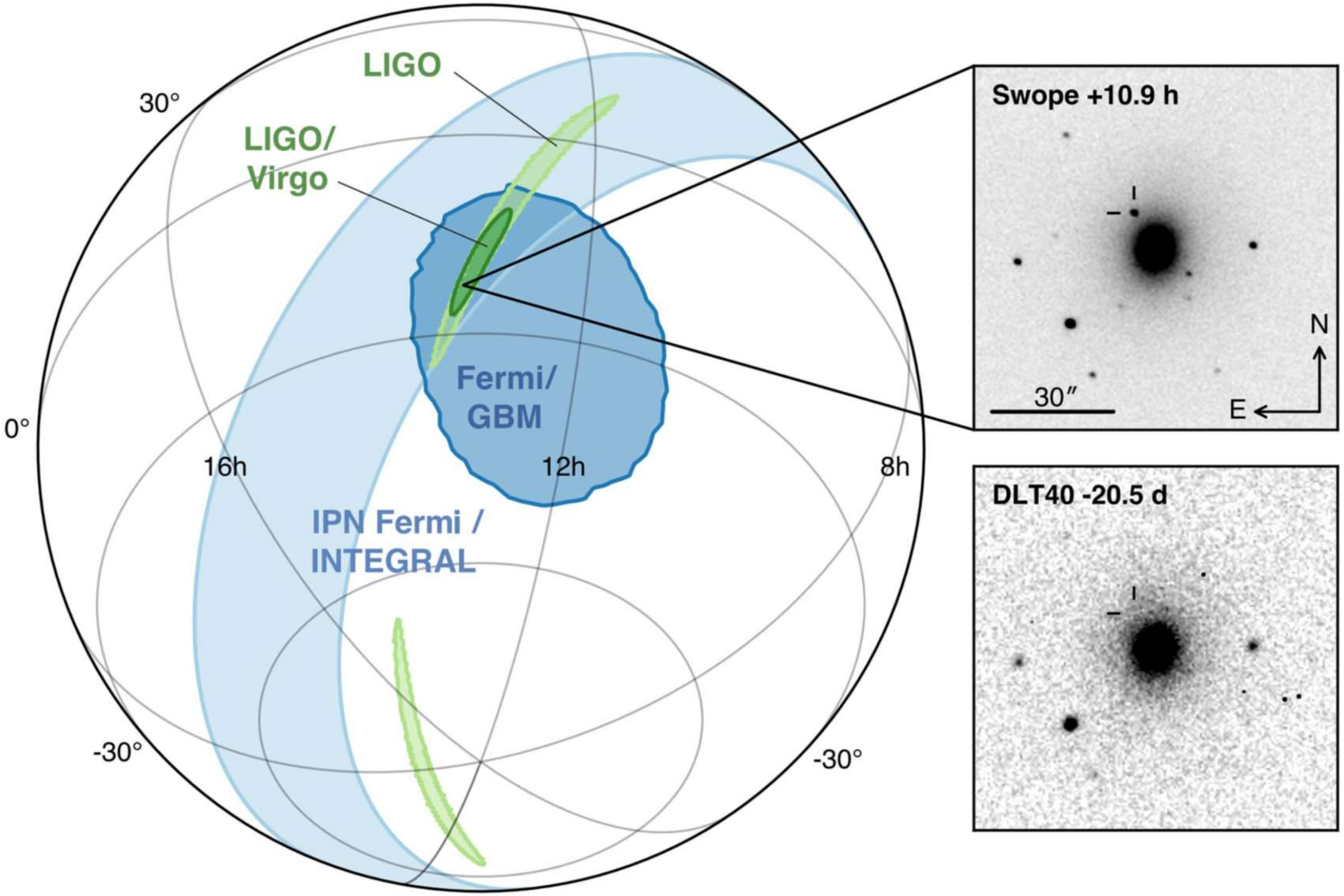


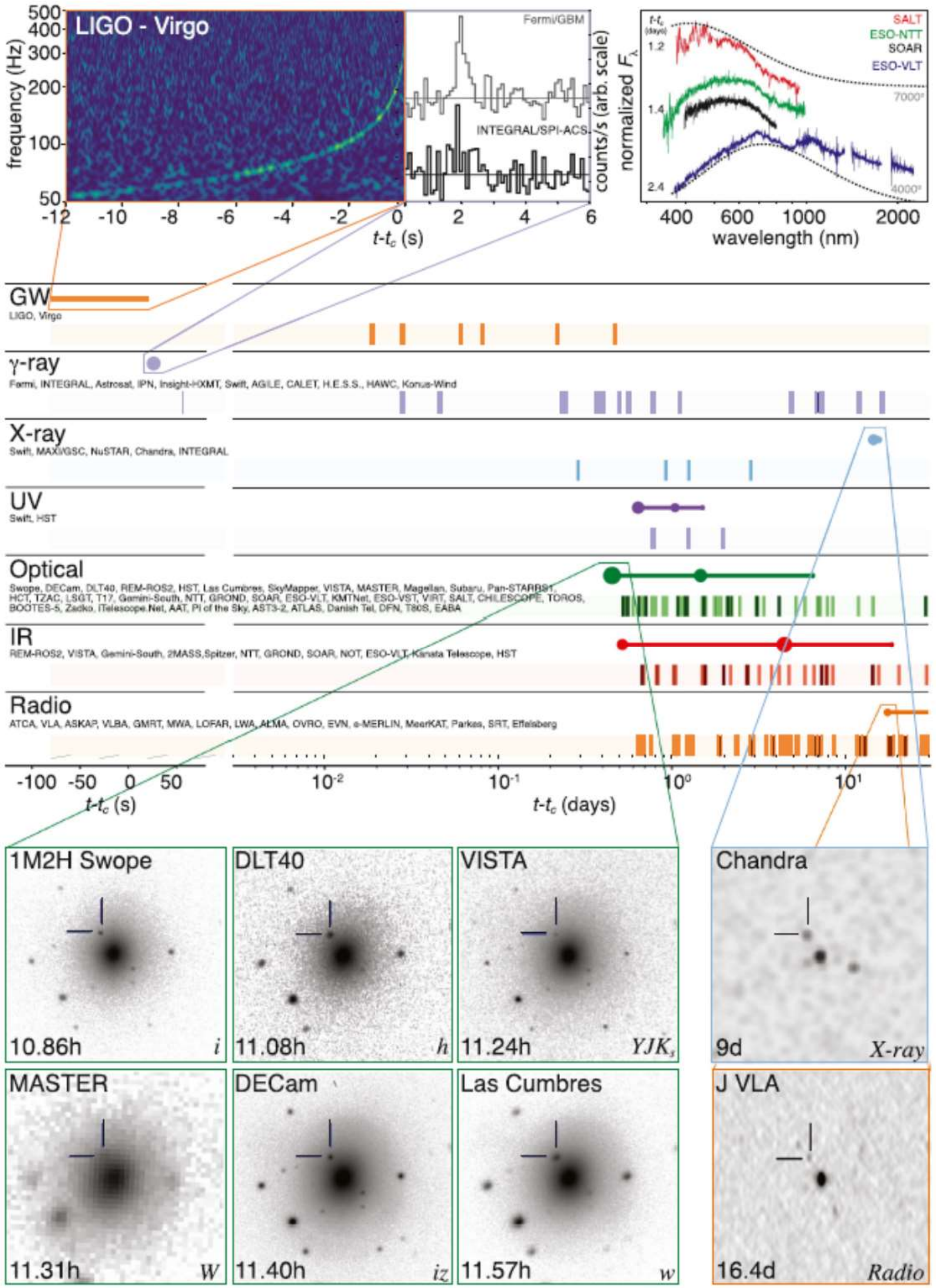
# Multi-Messenger Campaigns

GW150914



# The Birth of a New Field in Science

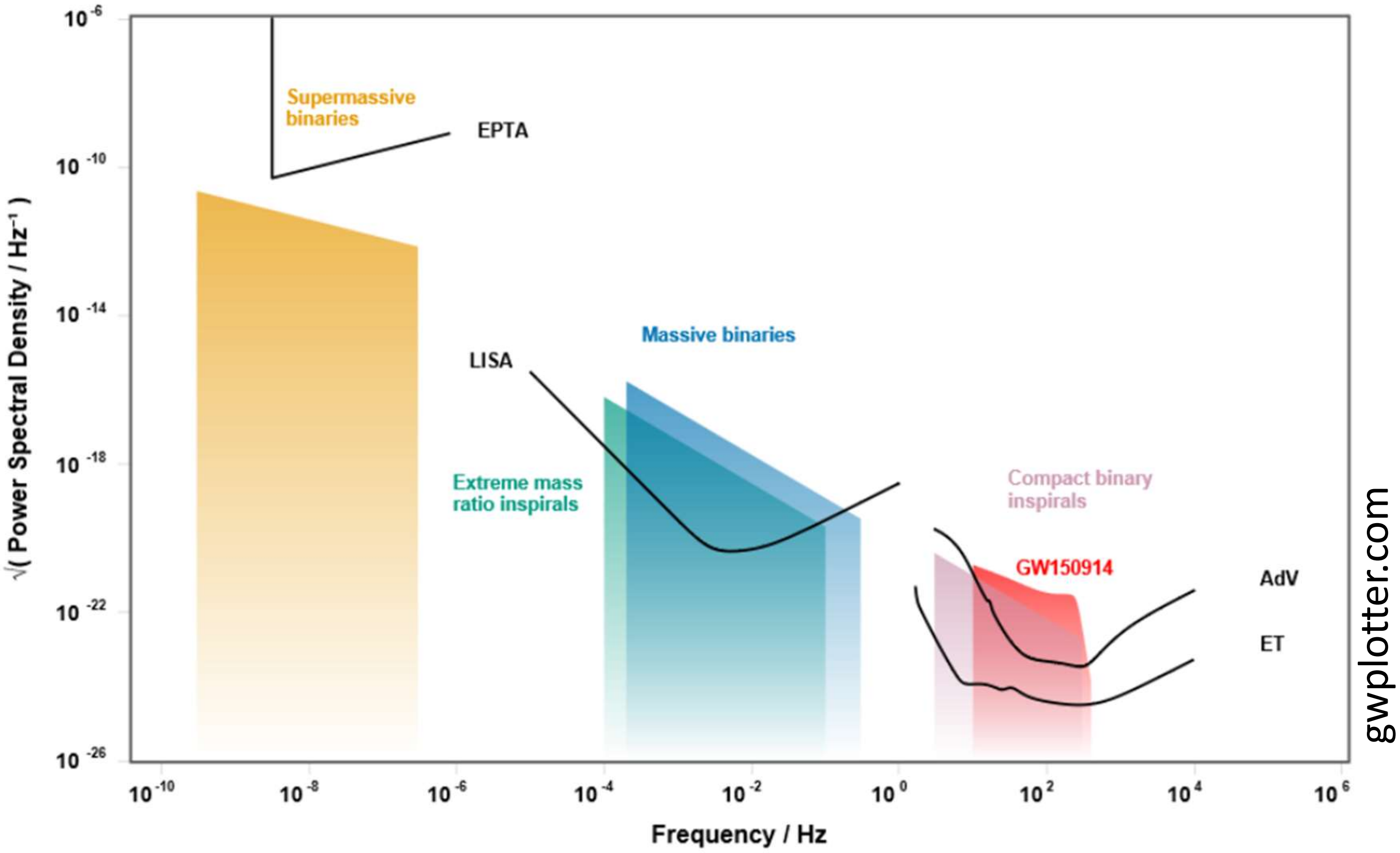




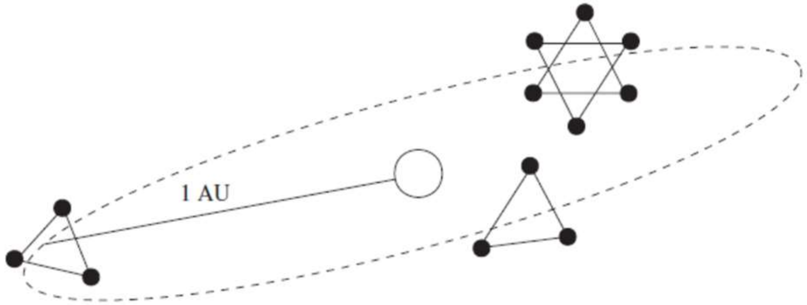

# GW170817

The largest scientific observation campaign in human history.

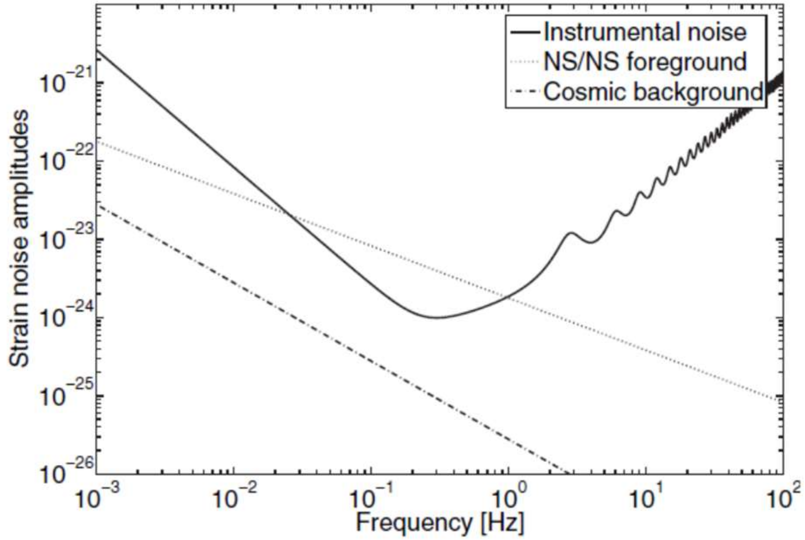
# Observation Bands



# The Big Bang Observer




BBO is formed by 4 LISA-type configurations for GW observations at 10mHz – 10Hz.




Distribution of detectors around Sun makes sure that all compact binaries are seen with high SNR.

The waveforms of all detected binaries need to be subtracted from the data streams.

A pair of collocated detectors is used for the final stochastic GW search.